Διαγώνισμα Άλγεβρας Α' Λυκείου 11/10/2020

Λύσεις

Θέμα Α

A1) Σχολικό Βιβλίο, σελίδα 15

A2) i) $(a-\beta)^2 = a^2 - 2a\beta + \beta^2$

ii) $(a+\beta+\gamma)^2 = a^2+\beta^2+\gamma^2 + 2a\beta + 2a\gamma + 2\beta\gamma$

iii) $a^3+\beta^3 = (a+\beta)(a^2 - a\beta + \beta^2)$

iv) $(a-\beta)^3 = a^3 - 3a^2\beta + 3a\beta^2 - \beta^3$

v) $-\beta^2 + a^2 = (a-\beta)(a+\beta)$

A3) Γ

A4) 1. Λ

2. Λ

3. Σ

4. Σ

5. Σ

Θέμα Β

B1) i) $(a-\beta)(a+\beta)(a^2+\beta^2) = (a^2-\beta^2)\cdot(a^2+\beta^2) =$

$= (a^2)^2 - (\beta^2)^2 =$

$= a^4 - \beta^4$

ii) $(a+\beta)^2 - (a-\beta)^2 = a^2 + 2a\beta + \beta^2 - (a^2 - 2a\beta + \beta^2 =$

$= a^2 + 2a\beta + \beta^2 - a^2 + 2a\beta - \beta^2 =$

$= 4a\beta$

B2) i) $\frac{x^2-4}{x^2-x} \cdot \frac{x^3-x}{x^2+2x} = \frac{(x-2)\cancel{(x+2)}}{\cancel{x}(x-1)} \cdot \frac{\cancel{x}(x^2-1)}{x\cancel{(x+2)}} =$

$= \frac{x-2}{\cancel{x-1}} \cdot \frac{\cancel{(x-1)}(x+1)}{x} =$

$= \frac{(x-2)(x+1)}{x}$

ii) $\dfrac{x^3-6x^2+9x}{x^3-9x} = \dfrac{\cancel{x}(x^2-6x+9)}{\cancel{x}(x^2-9)} = \dfrac{(x-3)^{\cancel{2}}}{(\cancel{x-3})(x+3)} = \dfrac{x-3}{x+3}$

iii) $\dfrac{x^2-x+2x-2}{x^2-1} = \dfrac{x(x-1)+2(x-1)}{(x-1)(x+1)} = \dfrac{(\cancel{x-1})(x+2)}{(\cancel{x-1})(x+1)} = \dfrac{x+2}{x+1}$

Θέμα Γ

Γ1 $\left[\left(1+\dfrac{x}{y}\right)\left(1+\dfrac{y}{x}\right)\right] : \left(x-\dfrac{y^2}{x}\right) = \left[\dfrac{y+x}{y} \cdot \dfrac{x+y}{x}\right] : \left(\dfrac{x^2-y^2}{x}\right) =$

$$= \frac{(x+y)^2}{xy} \cdot \frac{x}{x^2-y^2} =$$

$$= \frac{(x+y)^{\cancel{2}}}{\cancel{x}y} \cdot \frac{\cancel{x}}{(\cancel{x+y})(x-y)} =$$

$$= \frac{x+y}{y(x-y)}$$

Γ2 i) $\dfrac{3x^2+6x}{-x^2-4x-4} = \dfrac{3x(x+2)}{-(x^2+4x+4)} = \dfrac{3x(\cancel{x+2})}{-(x+2)^{\cancel{2}}} = -\dfrac{3x}{x+2}$

ii) $\dfrac{x^2+x}{x^2-4} \cdot \dfrac{x^2+5x+6}{x^2-1} = \dfrac{x(\cancel{x+1})}{(x-2)(x+2)} \cdot \dfrac{x^2+2x+3x+6}{(x-1)(\cancel{x+1})} =$

$$= \frac{x}{(x-2)(x+2)} \cdot \frac{x(x+2)+3(x+2)}{x-1} =$$

$$= \frac{x}{(x-2)(\cancel{x+2})} \cdot \frac{(\cancel{x+2})(x+3)}{x-1} =$$

$$= \frac{x(x+3)}{(x-2)(x-1)}$$

iii) $\left(x-\dfrac{1}{x}\right)^2 \cdot \dfrac{x^3+x^2}{(x+1)^3} = \left(\dfrac{x^2-1}{x}\right)^2 \cdot \dfrac{x^2\cdot(\cancel{x+1})}{(x+1)^{\cancel{3}2}} =$

$$= \left[\frac{(x-1)(x+1)}{x}\right]^2 \cdot \frac{x^2}{(x+1)^2} =$$

$$= \frac{(x-1)^2\cdot(\cancel{x+1})^2}{\cancel{x^2}} \cdot \frac{\cancel{x^2}}{(\cancel{x+1})^2} =$$

$$= (x-1)^2$$

iv) $\frac{x^2-9}{x^2-6x+9} : \left(\frac{x-3}{x+3}\right)^{-3} = \frac{(x-3)(x+3)}{(x-3)^2} : \left(\frac{x+3}{x-3}\right)^3 =$

$= \frac{x+3}{x-3} : \frac{(x+3)^3}{(x-3)^3} =$

$= \frac{\cancel{x+3}}{\cancel{x-3}} \cdot \frac{(x-3)^{\cancel{3}2}}{(x+3)^{\cancel{3}2}} =$

$= \frac{(x-3)^2}{(x+3)^2}$

Θέμα Δ

(Δ1) $A = \frac{\left(\frac{y^3}{x^2}\right)^4 : \left[\left(\frac{y^{-5}}{x^{-4}}\right)^3 \cdot (x^{-1} \cdot y^3)^{-5}\right]}{\left[\left(\frac{y^2}{x^3}\right)^{-4} \cdot \left(\frac{y}{x^3}\right)^3\right]^{-9}} = \frac{\frac{y^{12}}{x^8} : \left[\frac{y^{-15}}{x^{-12}} \cdot x^5 \cdot y^{-15}\right]}{\left[\frac{y^{-8}}{x^{-12}} \cdot \frac{y^3}{x^9}\right]^{-9}} =$

$= \frac{\frac{y^{12}}{x^8} : (y^{-30} \cdot x^{17})}{\left(\frac{y^{-5}}{x^{-3}}\right)^{-9}} = \frac{\frac{y^{12}}{x^8} \cdot \frac{1}{y^{-30} \cdot x^{17}}}{\frac{y^{45}}{x^{27}}} =$

$= \frac{\frac{y^{42}}{x^{25}}}{\frac{y^{45}}{x^{27}}} = \frac{x^{27} \cdot y^{42}}{x^{25} \cdot y^{45}} = x^2 \cdot y^{-3} = \frac{x^2}{y^3}$

(Δ2) i) Παρατηρώ ότι: $(3\alpha-\beta)+(3\beta-\gamma)+(3\gamma-\alpha) =$

$= 2\alpha+2\beta+2\gamma =$

$= 2 \cdot (\alpha+\beta+\gamma) =$

$= 0$

Οπότε η παράσταση παραγοντοποιείται:

$A = (3\alpha-\beta)^3+(3\beta-\gamma)^3+(3\gamma-\alpha)^3 =$

$= 3 \cdot (3\alpha-\beta) \cdot (3\beta-\gamma) \cdot (3\gamma-\alpha)$

ii) Απο την σχέση $\alpha+\beta+\gamma=0$ ισχύει $\gamma=-\alpha-\beta$

Οπότε $\gamma^2-\alpha^2-\beta^2 = (-\alpha-\beta)^2-\alpha^2-\beta^2 =$

$= [-(\alpha+\beta)]^2-\alpha^2-\beta^2 =$

$= (\alpha+\beta)^2-\alpha^2-\beta^2 =$

$= \cancel{\alpha^2}+2\alpha\beta+\cancel{\beta^2}-\cancel{\alpha^2}-\cancel{\beta^2} = 2\alpha\beta$

iii) $B = \dfrac{\alpha^3+\beta^3+\gamma^3-3\alpha\beta\cdot(\gamma-2)}{\gamma^2-\alpha^2-\beta^2} =$ (εφόσον $\alpha^3+\beta^3+\gamma^3=3\alpha\beta\gamma$)

$= \dfrac{3\alpha\beta\gamma-3\alpha\beta\gamma+6\alpha\beta}{2\alpha\beta} =$ (διότι από ερώτημα (ii) $\gamma^2-\alpha^2-\beta^2=2\alpha\beta$)

$= 3$

δηλαδή σταθερή.